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Ensemble Teleportation

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Abstract

The possibility of teleportation is by sure the most interesting consequence of quantum non-separability. So far, however, teleportation schemes have been formulated by use of state vectors and considering individual entities only. In the present article the feasibility of teleportation is examined on the basis of the rigorous ensemble interpretation of quantum mechanics (not to be confused with a mere treatment of noisy EPR pairs) leading to results which are unexpected from the usual point of view.

Key Words: Quantum teleportation, quantum non-separability, ensemble interpretation.
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1. Introduction

The possibility of teleportation is by sure the most interesting consequence of quantum non-separability. Bennett et al. were the first to assert that an "unknown quantum state $|\phi\rangle$ can be disassembled into, then later reconstructed from, purely classical information and purely nonclassical Einstein-Podolsky-Rosen (EPR) correlations. To do so, the sender, 'Alice,' and the receiver, 'Bob,' must prearrange the sharing of an EPR-correlated pair of particles. Alice makes a joint measurement on her EPR particle and the unknown quantum system, and sends Bob the classical result. Knowing this, Bob can convert the state of his EPR particle into an exact replica of the unknown state which Alice destroyed." (Quoted from the abstract of [1]. Note that in this teleportation scheme no matter is shifted spatially. Instead, a quantum-mechanical *state* is transferred from one material carrier to another.) Meanwhile several experiments have been successfully performed by different research groups, demonstrating the feasibility of teleportation [2–7] With the exception of [7], all experiments employ photons both for the generation of the EPR pair and to materialize the unknown state. There are proposals to realize teleportation using massive entities, i.e., atoms in high-Q cavities [8–11], solid-state systems [12], and clouds of atoms [13, 14]. Recently the first corresponding experiments have been successfully performed by using trapped Ca [15] and Be ions [16].

A crucial point of every experimental implementation of a theoretical teleportation scheme is the joint measurement of Alice's half of the EPR pair and the unknown entity (the state to be teleported [17]). At present new ideas are discussed to overcome these difficulties [18, 19, 20], and numerous researchers investigate the teleportation fidelity if non-ideal EPR pairs are used (see, e.g., [21–24]).

So far teleportation schemes are formulated mostly by use of state vectors and considering individual entities only. In previous articles the present author has shown that an *ensemble* interpretation of quantum mechanics (QM) is useful to unveil precisely the actual core of EPR's discovery [25, 26, 27] which is the non-separability of the quantum world. So it is a challenging task to analyze teleportation in the frame of said ensemble interpretation as well, especially since some of the proposals mentioned above seem to make explicit use of ensembles. [In the following we do not make use of the ergodic hypothesis. Instead, if speaking about an ensemble, it is understood as a *material* ensemble, i.e., a set of sufficiently many interaction-free and identical particles to allow for a law of large number to be valid for all practical purposes.] Therefore statistical operators are used instead of state vectors, because they reflect directly the ensemble way of viewing QM. Recall that the founding fathers of QM were convinced that the state vectors refer to individual entities only.

In this article we will *not* make use of the so-called simple ensemble interpretation, which rests on the idea that each individual member of the ensemble always *has* (in the sense of EPR's principle of reality) precise values (properties) for all its property types. In the case of a certain quantum system, Gillespie proved that this interpretation is not possible [28]. This result, however, is not of relevance for the approach employed in the present article, because here no statements regarding properties to be ascribed to individual entities are made. Only the whole, i.e., the ensemble, will be considered as an element of QM. This *rigorous* ensemble interpretation completely ignores the individual and asserts that QM makes predicitons about ensembles only. The being of the individual remains veiled.

It should be clear that ensembles in the sense described above have nothing in common with the so-called noisy EPR pairs often discussed in the context of non-ideal teleportation.

2. The teleportation process

2.1. Preliminary remarks

Basis of all following considerations is the *rigorous* ensemble interpretation of QM mentioned above, i.e., it is presupposed that QM makes

- statistical statements on
- the results of measurements on
- ensembles

only. This approach offers several advantages in the handling of non-separability and quantum holism. First of all, it virtually demands the use of statistical operators whereby we may ignore the notion of a state vector and all the questions regarding its meaning which have recently been discussed in detail by Laloë [29]. In consequence, all problems connected with the reduction postulate are removed as well. Moreover, we are not concerned about the reality of individual microentities. Even the whole discussion about reality itself becomes irrelevant. But this interpretation still offers further advantages: It reflects the practical situation of the experiments performed insofar as it deals only with ensembles of entities and not with single constituents. No experiment has been published where less than some thousand single runs have been done. And finally, also the problematic notion of the primary probability of a single event (Popper's propensity) is avoided. Here we speak about probabilities in the quite trivial sense of relative frequencies only. Details can be found in [25, 26, 27].

A given ensemble of micro-entities is represented by a self-adjoint operator ρ, the so-called statistical operator, with $\mathrm{Tr}(\rho) = 1$ and positive spectrum (including 0). Now let this ρ be defined on a 2^3 dimensional Hilbert space $\mathcal{H}_{\text{total}} = \mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}} \otimes \mathcal{H}_{\mathrm{C}}$ where the set $\{|\alpha_1\rangle, |\alpha_2\rangle\}$ forms an orthonormal basis of the subspace $\mathcal{H}_{\mathrm{A}}$, and let respective basis sets be given analogously for the other subspaces. In the four dimensional subspace $\mathcal{H}_\mathrm{A} \otimes \mathcal{H}_\mathrm{B}$ we generate a Bell-type basis according to the definition

$$|\Psi_\mathrm{e}^\pm\rangle := \frac{1}{\sqrt{2}} \left(|\alpha_1\rangle|\beta_1\rangle \pm |\alpha_2\rangle|\beta_2\rangle\right) \tag{1}$$

$$|\Psi_\mathrm{o}^\pm\rangle := \frac{1}{\sqrt{2}} \left(|\alpha_1\rangle|\beta_2\rangle \pm |\alpha_2\rangle|\beta_1\rangle\right). \tag{2}$$

With the aid of this Bell-basis, four statistical operators can be defined:

$$\begin{aligned}\rho_{1,2} &:= |\Psi_\mathrm{e}^\pm\rangle\langle\Psi_\mathrm{e}^\pm| \\ &= \frac{1}{2}\left(\hat{A}_{11}\otimes\hat{B}_{11} \pm \hat{A}_{12}\otimes\hat{B}_{12} \pm \hat{A}_{21}\otimes\hat{B}_{21} + \hat{A}_{22}\otimes\hat{B}_{22}\right); \qquad (3)\\ \rho_{3,4} &:= |\Psi_\mathrm{o}^\pm\rangle\langle\Psi_\mathrm{o}^\pm| \\ &= \frac{1}{2}\left(\hat{A}_{11}\otimes\hat{B}_{22} \pm \hat{A}_{12}\otimes\hat{B}_{21} \pm \hat{A}_{21}\otimes\hat{B}_{12} + \hat{A}_{22}\otimes\hat{B}_{11}\right), \qquad (4)\end{aligned}$$

where $\hat{A}_{ij} = |\alpha_i\rangle\langle\alpha_j|$ and $\hat{B}_{ij} = |\beta_i\rangle\langle\beta_j|$. For these four operators the following statements are valid:

- $\rho_i^2 = \rho_i$
- $\rho_i\rho_j = \hat{0}\ \forall\, i \neq j$
- They are *non*-separable [26].

2.2. A teleportation scheme for ensembles

2.2.1. Formalism

Suppose we are in possession of a generator producing an ensemble $\{(\mathrm{AB})_i\}$ of micro-entities $(\mathrm{AB})_i$, each of them dissociating according to $(\mathrm{AB})_i \longrightarrow \mathrm{A}_i + \mathrm{B}_i$. The sub-ensemble of all A_i is sent either one by one or as a whole to an observer named Alice whereas the sub-ensemble of all B_i is sent correspondingly to a second observer named Bob. [The problem of storing ensembles is discussed in section 3.] The ensemble $\{\{\mathrm{A}_i\},\{\mathrm{B}_i\}\}$ consisting of the two sub-ensembles is represented, say, by the statistical operator ρ_4 (see equation (4)) which means that we consider the ensemble in a *pure* state.

Now Alice obtains a further entity-ensemble, $\{\mathrm{C}_i\}$, which is assumed to be in a pure state (ρ_C) as well. Then the new total ensemble $\{\{\mathrm{C}_i\},\{\mathrm{A}_i\},\{\mathrm{B}_i\}\}$ is represented by

$$\rho_\mathrm{total} = \rho_\mathrm{C} \otimes \rho_4, \tag{5}$$

where ρ_C is given by

$$\rho_\mathrm{C} = \sum_{k,l=1}^{2} c_{kl}\hat{C}_{kl}; \tag{6}$$

and $\mathrm{Tr}(\rho_\mathrm{C}) = 1$. The actual values of the coefficients c_{kl} must not be known to Alice.

$$\begin{aligned}\Rightarrow \rho_\mathrm{total} &= \frac{1}{2}\left(c_{11}\hat{C}_{11} + c_{12}\hat{C}_{12} + c_{21}\hat{C}_{21} + c_{22}\hat{C}_{22}\right) \\ &\quad \otimes \left(\hat{A}_{11}\otimes\hat{B}_{22} - \hat{A}_{12}\otimes\hat{B}_{21} - \hat{A}_{21}\otimes\hat{B}_{12} + \hat{A}_{22}\otimes\hat{B}_{11}\right). \qquad (7)\end{aligned}$$

We define four new statistical operators, ρ_1', ρ_2', ρ_3', and ρ_4', so that they are analogous to the already introduced operators ρ_1, ρ_2, ρ_3, and ρ_4, respectively. The primed operators, however, shall act on $\mathcal{H}_\mathrm{C} \otimes \mathcal{H}_\mathrm{A}$

instead of $\mathcal{H}_A \otimes \mathcal{H}_B$. With the aid of these new operators, and after some lengthy but straightforward manipulations, ρ_{total} can be brought into the form

$$\begin{aligned} 2\rho_{\text{total}} &= \rho_1' \otimes (c_{11}\hat{B}_{22} - c_{12}\hat{B}_{21} - c_{21}\hat{B}_{12} + c_{22}\hat{B}_{11}) \\ &+ \rho_2' \otimes (c_{11}\hat{B}_{22} + c_{12}\hat{B}_{21} + c_{21}\hat{B}_{12} + c_{22}\hat{B}_{11}) \\ &+ \rho_3' \otimes (c_{11}\hat{B}_{11} - c_{12}\hat{B}_{12} - c_{21}\hat{B}_{21} + c_{22}\hat{B}_{22}) \\ &+ \rho_4' \otimes (c_{11}\hat{B}_{11} + c_{12}\hat{B}_{12} + c_{21}\hat{B}_{21} + c_{22}\hat{B}_{22}) \\ &+ [\ldots], \end{aligned} \tag{8}$$

where

$$[\ldots] = \sum_{i,j=1}^{2} \hat{F}_{ij} \tag{9}$$

and

$$\begin{aligned} \hat{F}_{11} &= -(c_{22}\hat{A}_{11} \otimes \hat{B}_{11} + c_{11}\hat{A}_{12} \otimes \hat{B}_{21} \\ &\quad + c_{11}\hat{A}_{21} \otimes \hat{B}_{12} + c_{22}\hat{A}_{22} \otimes \hat{B}_{22}) \otimes \hat{C}_{11} \end{aligned} \tag{10}$$

$$\begin{aligned} \hat{F}_{12} &= (c_{12}\hat{A}_{11} \otimes \hat{B}_{22} + c_{21}\hat{A}_{12} \otimes \hat{B}_{12} \\ &\quad + c_{21}\hat{A}_{21} \otimes \hat{B}_{21} + c_{12}\hat{A}_{22} \otimes \hat{B}_{11}) \otimes \hat{C}_{12} \end{aligned} \tag{11}$$

$$\begin{aligned} \hat{F}_{21} &= (c_{21}\hat{A}_{11} \otimes \hat{B}_{22} + c_{12}\hat{A}_{12} \otimes \hat{B}_{12} \\ &\quad + c_{12}\hat{A}_{21} \otimes \hat{B}_{21} + c_{21}\hat{A}_{22} \otimes \hat{B}_{11}) \otimes \hat{C}_{21} \end{aligned} \tag{12}$$

$$\begin{aligned} \hat{F}_{22} &= -(c_{11}\hat{A}_{11} \otimes \hat{B}_{11} + c_{22}\hat{A}_{12} \otimes \hat{B}_{21} \\ &\quad + c_{22}\hat{A}_{21} \otimes \hat{B}_{12} + c_{11}\hat{A}_{22} \otimes \hat{B}_{22}) \otimes \hat{C}_{22}. \end{aligned} \tag{13}$$

By use of

$$\rho_B = c_{11}\hat{B}_{11} + c_{12}\hat{B}_{12} + c_{21}\hat{B}_{21} + c_{22}\hat{B}_{22} \tag{14}$$

and employing the Pauli matrices σ_1 and σ_3, both related to the subspace $\mathcal{H}_B$, equation (8) can be written as

$$\begin{aligned} 2\rho_{\text{total}} &= \rho_1' \otimes (\sigma_3\sigma_1\rho_B\sigma_1\sigma_3) + \rho_2' \otimes (\sigma_1\rho_B\sigma_1) + \rho_3' \otimes (\sigma_3\rho_B\sigma_3) \\ &+ \rho_4' \otimes \rho_B + [\ldots]. \end{aligned} \tag{15}$$

In the usual teleportation scheme formulated on the basis of wavefunctions of individual objects, Alice measures by which of the four states $|\Phi_e^\pm\rangle = 1/\sqrt{2}\ (|\gamma_1\rangle|\alpha_1\rangle \pm |\gamma_2\rangle|\alpha_2\rangle)$ and $|\Phi_o^\pm\rangle = 1/\sqrt{2}\ (|\gamma_1\rangle|\alpha_2\rangle \pm |\gamma_2\rangle|\alpha_1\rangle)$, respectively, her combined system C+A is represented. Recall, however, that this operation changes the state of the total system C+A+B too. After the measurement Alice informs Bob by use of a classical transmission channel about the result so that Bob knows what to do in order to transform the state of his B into the original state of C. Said information consists of two bits. In this way a state can be teleported, but in contrast to this scheme we will see that in the case of ensembles things may be quite different.

2.2.2. Interpretation

Consider that Alice *changes* the hitherto unknown state of C+A rather than measuring it. For example Alice can apply one of four possible preparation tools to influence the state of C+A. Assume for the moment that Alice's operation on her sub-ensemble C+A consists in the projection onto ρ_1', i.e., she acts upon the total system C+A+B by use of the operator $\rho'_1 \otimes \hat{1}_{\mathrm{B}}$. [The experimental realizability is discussed in Section 3.] Then the "expectation value" $\langle \rho'_1 \otimes \hat{1}_{\mathrm{B}} \rangle$ of her activity is the partial trace

$$\mathrm{Tr}_{\mathrm{C,A}}\left((\rho'_1 \otimes \hat{1}_{\mathrm{B}})\rho_{\mathrm{total}}\right) \tag{16}$$

(see Appendix) describing the state of B conditioned by ρ'_1 realized on C + A. Due to the mutual orthogonality of the primed operators, we obtain

$$\begin{aligned} \mathrm{Tr}_{\mathrm{C,A}}\left((\rho'_1 \otimes \hat{1}_{\mathrm{B}})\rho_{\mathrm{total}}\right) &= \frac{1}{4}\left(-c_{22}\hat{B}_{11} + c_{21}\hat{B}_{12} + c_{12}\hat{B}_{21} - c_{11}\hat{B}_{22}\right) \\ &\quad + \frac{1}{2}\,\sigma_3\sigma_1\rho_{\mathrm{B}}\sigma_1\sigma_3. \end{aligned} \tag{17}$$

Insertion of ρ_{B} from equation (14) yields

$$\sigma_3\sigma_1\rho_{\mathrm{B}}\sigma_1\sigma_3 = c_{22}\hat{B}_{11} - c_{21}\hat{B}_{12} - c_{12}\hat{B}_{21} + c_{11}\hat{B}_{22}; \tag{18}$$

$$\Rightarrow \mathrm{Tr}_{\mathrm{C,A}}\left((\rho'_1 \otimes \hat{1}_{\mathrm{B}})\rho_{\mathrm{total}}\right) = \frac{1}{4}\,\sigma_3\sigma_1\rho_{\mathrm{B}}\sigma_1\sigma_3. \tag{19}$$

Therefore Alice's "expectation value" is an operator which refers to Bob's sub-ensemble. It is seen immediately that this operator has a trace of 1/4, i.e., it must be re-normalized by division by its norm

$$\Rightarrow \tilde{\mathrm{Tr}}_{\mathrm{C,A}}\left((\rho'_1 \otimes \hat{1}_{\mathrm{B}})\rho_{\mathrm{total}}\right) = \sigma_3\sigma_1\rho_{\mathrm{B}}\sigma_1\sigma_3. \tag{20}$$

This re-normalization does *not* imply any post-selection of ensemble constituents. No particle will be omitted. Bob only adjusts the descriptive entity of B.

Now the expression on the right side of (20) is a statistical operator representing the state of Bob's sub-ensemble after the preparation of Alice. Recall that every influence exerted on A automatically changes B as well, because the sub-ensembles A and B are in a non-separable state. So (20) describes what Bob has at hand. Now Alice must tell Bob *what* she has done, i.e., if, before the beginning of the whole experiment, Alice and Bob have agreed upon both the allowed preparations on Alice's side and a corresponding number code, then Alice has to send two bits of classical information to Bob, and he will be able to apply some operations so that his sub-ensemble B is represented by ρ_{B} which is the analog of the unknown ρ_{C} in Bob's Hilbert space. In this way teleportation of a statistical operator is achieved; but recall that Alice must inform Bob about what she has *done*, not about the *result* of her action, i.e., Alice does not read any pointer position. She makes a pure preparation without any gain or loss of information regarding the ensemble! The only thing one has to require is that the operator representing what Alice does must be physically realizable.

One could object that, in the case of ensembles, teleportation looses its unique quantum features because Alice simply could sacrifice some of the identical constituents of the C-ensemble to determine ρ_{C} (equation (6)) and then transmit the information to Bob via a classical channel. She could, at least in principle, perform a measurement even of the *complete* C-ensemble, therefore obtaining *precise* values for all coefficients c_{kl} and, by transmitting a nevertheless finite number of bits, enable Bob to transform his B-sub-ensemble accordingly. This is trivial indeed. The point, however, is that Alice does not need to know the c_{kl}. She manipulates the C+A-ensemble and, without knowing anything about C, still enables Bob to recreate it. And this is certainly non-trivial.

So far ensemble teleportation and the teleportation of individual micro-entities along the usual scheme seem to be equivalent, although the role of Alice is different, and in fact, after application of the unitary transformations $\sigma_3 \ldots \sigma_3$ and $\sigma_1 \ldots \sigma_1$, Bob ends up with the desired result.

2.3. Ensemble teleportation by use of one bit of information only

In the most general case Alice's preparation consists in the application of the operator

$$\hat{P} = \sum_{k,l,m,n} u_{klmn} |\gamma_k\rangle\langle\gamma_l| \otimes |\alpha_m\rangle\langle\alpha_n| \equiv \sum_{k,l,m,n} u_{klmn}\, \hat{C}_{kl} \otimes \hat{A}_{mn}. \tag{21}$$

With ρ_{total} from equation (7) and using the fact that, e.g., $\hat{C}_{kl}\hat{C}_{pq} = \hat{C}_{kq}$ if $l = p$ and $\hat{0}$ otherwise, we then obtain:

$$\begin{aligned}(\hat{P} \otimes \hat{1}_{\mathrm{B}})\rho_{\mathrm{total}} &= \frac{1}{2} \sum_{k,m,p,q} c_{pq}\hat{C}_{kq} \otimes \Big(\hat{A}_{m1} \otimes (u_{kpm1}\hat{B}_{22} - u_{kpm2}\hat{B}_{12}) \\ &\quad + \hat{A}_{m2} \otimes (-u_{kpm1}\hat{B}_{21} + u_{kpm2}\hat{B}_{11}) \Big). \end{aligned} \tag{22}$$

Again we form the trace with regard to both $\mathcal{H}_{\mathrm{C}}$ and $\mathcal{H}_{\mathrm{A}}$:

$$\mathrm{Tr}_{\mathrm{C,A}} \Big((\hat{P} \otimes \hat{1}_{\mathrm{B}})\rho_{\mathrm{total}} \Big) = \frac{1}{2} \sum_{i,p} c_{pi}(u_{ip11}\hat{B}_{22} - u_{ip12}\hat{B}_{12} - u_{ip21}\hat{B}_{21} + u_{ip22}\hat{B}_{11}). \tag{23}$$

This equation defines an operator, called ρ_{Bob}, which, after re-normalization, is the final result of Alice's manipulation on her ensemble $\{\{\mathrm{C}_i\},\{\mathrm{A}_i\}\}$:

$$\tilde{\rho}_{\mathrm{Bob}} = \frac{\rho_{\mathrm{Bob}}}{\mathrm{Tr}(\rho_{\mathrm{Bob}})} \tag{24}$$

ρ_{Bob} represents the essence of the actual state of Bob's sub-ensemble. In the basis of the operators $\hat{B}_{ij}$ it can be written vectorially as

$$\rho_{\mathrm{Bob}} = \frac{1}{2} \begin{pmatrix} c_{11}u_{1122} + c_{12}u_{2122} + c_{21}u_{1222} + c_{22}u_{2222} \\ -c_{11}u_{1112} - c_{12}u_{2112} - c_{21}u_{1212} - c_{22}u_{2212} \\ -c_{11}u_{1121} - c_{12}u_{2121} - c_{21}u_{1221} - c_{22}u_{2221} \\ c_{11}u_{1111} + c_{12}u_{2111} + c_{21}u_{1211} + c_{22}u_{2211} \end{pmatrix}. \tag{25}$$

This vector results from the original vector $\vec{c} = (c_{11}, c_{12}, c_{21}, c_{22})$ (see equation (6)) by the transformation

$$\mathbf{T} = \begin{pmatrix} u_{1122} & u_{2122} & u_{1222} & u_{2222} \\ -u_{1112} & -u_{2112} & -u_{1212} & -u_{2212} \\ -u_{1121} & -u_{2121} & -u_{1221} & -u_{2221} \\ u_{1111} & u_{2111} & u_{1211} & u_{2211} \end{pmatrix}, \tag{26}$$

i.e.,

$$\tilde{\rho}_{\mathrm{Bob}} = \frac{\frac{1}{2}\mathbf{T}\vec{c}}{\|\frac{1}{2}\mathbf{T}\vec{c}\|}. \tag{27}$$

So Bob can impress the original state $\vec{c}$ on his sub-ensemble by applying the inverse transformation. Here it is presupposed that the inverse actually exists which is always the case if Alice projects on one of the ρ'_i.

But what would happen if Bob completely renounced any manipulation of his $\tilde{\rho}_{\mathrm{Bob}}$, i.e., if he would *not* apply the inverse transformation? We start the investigation of this case by rewriting Bob's statistical operator in the form

$$\tilde{\rho}_{\mathrm{Bob}} = \vec{c} + \left(\frac{\mathbf{T}}{\|\mathbf{T}\vec{c}\|} - \mathbf{1} \right) \vec{c} \tag{28}$$

where the second term represents the contamination of $\vec{c}$ due to the fact that Bob has not done anything at all. The success of a teleportation, the so-called fidelity f, is given by

$$f = \mathrm{Tr}(\rho_{\mathrm{C}}|_{(\text{ensemble B})} \times \tilde{\rho}_{\mathrm{Bob}}). \tag{29}$$

In terms of our matrix representation and by use of (28) this definition reduces to the sum of two scalar products:

$$f = \underbrace{\vec{c}\,\vec{c}}_{=1} + \vec{c}\left(\frac{\mathbf{T}}{\|\mathbf{T}\vec{c}\|} - \mathbf{1}\right)\vec{c}. \tag{30}$$

Now assume that Alice had performed a projection onto ρ'_1.

$$\Rightarrow \mathbf{T} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \tag{31}$$

$$\begin{aligned} \Rightarrow \|\mathbf{T}\vec{c}\| &= \frac{1}{2} && (32)\\ \Rightarrow f &= 2\,\vec{c}\mathbf{T}\vec{c} && (33)\\ &= 2\,c_{11}(1-c_{11})\,(1-\cos 2\phi), && (34) \end{aligned}$$

i.e., the fidelity becomes phase dependent. It attains its maximum if $\phi = \pi/2$ and $c_{11} = 1/2$. $\Rightarrow f_{max} = 1$. But recall that the incoming state is unknown so that also a fidelity of 0 might be obtained (if, e.g., $\phi = 0$). In the end, Bob's inactivity does not pay off.

Therefore Bob must find ways to manipulate his sub-ensemble. Note that the average fidelity over all possible inputs has been determined, based on the usual teleportation scheme, to be 2/3 [30]. Alice, however, is in the position of being able to save some of Bob's efforts. Assume that Alice has already made up her mind about the details of her operation well before the beginning of the experiment, and assume further that she has told Bob what she wants to do. In this case Alice needs to send a single bit of classical information only to inform Bob that her part of the experiment has taken place. Strictly speaking, this bit can be reduced even to a single "click" ($\equiv$ yes, it's done). But this idea can yet be carried on. If Alice and Bob, at the very beginning of the whole story, already achieve agreement on the *time* when the preparation shall be performed, then Bob has to wait at that time only and he will then be able to reproduce $\vec{c}$ without obtaining any information from Alice at all. Note, however, that this idea also applies to the usual teleportation scheme.

2.4. Ensemble teleportation without any action on Bob's side

A surprising result is found by choosing

$$\mathbf{T} = \alpha\mathbf{1} \tag{35}$$

with $\alpha > 0$, because this immediately yields $\tilde{\rho}_{\mathrm{Bob}} = \vec{c}$ (in the basis of the $\hat{B}_{ij}$), i.e., in this case Bob would *have* the teleported state of $\{\mathrm{C}_i\}$ after Alice's preparation *without doing anything at all*! This situation raises the following question: Can the condition (35) imposed on $\mathbf{T}$ be realized by an operator $\hat{P}$? Then Alice would be able to prepare the total ensemble in a way that the *unknown* state $\vec{c}$ is teleported *automatically.*

From relations (35), (26), and (21) the necessary operator for automatic teleportation is given by

$$\hat{P}_{\mathrm{aut}} = \alpha\left(\hat{C}_{11}\otimes\hat{A}_{22} - \hat{C}_{21}\otimes\hat{A}_{12} - \hat{C}_{12}\otimes\hat{A}_{21} + \hat{C}_{22}\otimes\hat{A}_{11}\right). \tag{36}$$

The comparison of this equation and (4) shows that $\hat{P}_{\rm aut} = \rho'_4$ if $\alpha = 1/2$, i.e., also $\hat{P}_{\rm aut}$ is self-adjoint and its eigenvalues are $\lambda_{1,2,3} = 0$ and $\lambda_4 = +1$. Proceeding in the same way as in the previous subsection we then obtain $\tilde{\rho}_{\rm Bob} = \vec{c}$, i.e., complete ensemble teleportation is possible even if the second observer does nothing.

In summary: Let Alice's measurement or operation be described by the operator $\hat{P}_{\rm aut}$. After her manipulation of the C-A-ensemble Bob's B-sub-ensemble is in the state (28), i.e., in this case Bob's B-sub-ensemble would be in the desired state just after Alice's measurement—*without* any action on Bob's side (as, e.g., selecting constituents of his sub-ensemble) and *without* any information to be transmitted. At most, Alice could send one single 'ping' if she has done her work so that Bob will know that his sub-ensemble is now in the state described by $\vec{c}$. But that is all. The only open question is how the operation given by $\hat{P}_{\rm aut}$ can be realized in the lab. This question will be addressed in the following section.

This kind of teleportation of course has nothing to do with any mysterious effect as, e.g., superluminal communication, and no paradoxes arise. The fundament of any EPR correlation is the fact that neither of the two sub-ensembles has an independent existence of its own. Each of them remembers the common origin and is, therefore, in the possession of the *full* information. This point is explained in detail in [25, 27].

In the case of automatic teleportation ($\mathbf{T} = \alpha\mathbf{1}$) relation (30) shows that the fidelity is equal to 1 irrespective of the incoming state $\vec{c}$. So an arbitrary incoming state can not only be transported without any action on Bob's side but also with maximum fidelity. The only information which has to be transmitted is a classical 'ping' to indicate that Alice has done what she had to do.

It should be mentioned that this proposal has nothing in common with what happens in the original experiment of the Zeilinger group [2]. In said experiment a projection onto one of the four Bell states has been realized insofar as a corresponding sub-ensemble was selected out of the original total ensemble. For this sub-ensemble, representing one quarter of the original constituents only, automatic teleportation was achieved. This approach, however, is totally different from what is proposed here. *No* out-selection has to be performed. The *whole* ensemble is present in the teleportation scheme.

3. Discussion

- How do Alice and Bob perform their measurements/preparations? With individual entities this question is trivial, but how can *ensemble* measurements be realized?

If Alice obtains the constituents of her ensemble $\{\{\mathrm{C}_i\},\{\mathrm{A}_i\}\}$ one by one, i.e., one pair C_i+A_i at a time, then her activities do not differ from the usual case with the only exception that she has to process the whole ensemble until $\rho_{\rm Bob}$ is available. But also this is not a real restriction, because experimentalists always rely on a vast amount of single runs and not only on one. The problem, however, is that Bob has to store his ensemble for the time Alice needs to produce $\rho_{\rm Bob}$. Assume that the ensemble $\{\{\mathrm{A}_i\},\{\mathrm{B}_i\}\}$ used to teleport the state of $\{\mathrm{C}_i\}$ consists of atoms or molecules. They can be kept, e.g., in an electromagnetic trap, but the field induces an interaction between the single entities so that Bob's $\{\mathrm{B}_i\}$ is more a multi-particle system than an ensemble in the strict sense, and it is to be expected that the interaction will influence the teleportation fidelity significantly. If, on the other hand, the B_i are stored in a vessel, then this gas must be diluted so far that both dipolar and van der Waals interaction can be neglected and that during the storage time the collision probability is negligibly small.

It has, however, been shown [27] that the EPR correlations are *contextual*. So, if the ensemble constituents interact regarding *another* property type than the correlation is established, then storing in clouds or as a gas in a vessel becomes feasible. For example: If a molecule M_2 with point group symmetry $\mathrm{C_i}$ dissociates into two chiral fragments M with opposite handedness, then it is to be expected that the fragments are EPR correlated (see [31] for a detailed description of this experiment), i.e., the correlation is established by chirality. This correlation is extremely stable against external fields and thermal collisions so that an interaction caused by the storage medium will not lead to decoherence (at least for the time necessary to store the ensemble). [This means that also the ensemble property is contextual. From a certain point of view a cloud of atoms is to be considered a multi-particle system whereas from another point of view it can behave as an ensemble in the strict sense.]

- How to perform the required operation leading to the automatic impression of the unknown state $\vec{c}$ onto Bob's sub-ensemble?

As long as an operation is unitary it is always possible to realize it experimentally, at least if photons are used [32]. $\hat{P}_{\text{aut}}$, however, is self-adjoint and can therefore be approximated by a complicated sum of products of unitary operators and the unit operator, because every self-adjoint operator $\hat{A}$ acts as the generator of a unitary operator $\exp(i\hat{A})$.

4. Summary

The feasibility of teleportation is examined from an ensemble point of view, and it is shown that it is possible to teleport the state of an ensemble which is given by its statistical operator. While the sender has to apply physical realizations of projection operators the necessary operations on the side of the receiver are in general unitary.

It is also shown that teleportation allows for total abdication of any A POSTERIORI information exchange between sender and receiver if they fix prior to the start of the experiment what Alice will do and when it shall happen.

For arbitrary pure incoming states complete ensemble teleportation is possible even if the receiver does not do anything with his sub-ensemble at all. In order to achieve said automatic teleportation the sender has to apply the physical realization of a special self-adjoint operator. The fidelity of this process amounts to 1.

Future work will i) investigate the influence of suboptimal conditions on the fidelity and ii) elucidate whether the *classical* teleportation scheme of Cerf et al. [33] can be reformulated in the present ensemble approach as well.

Appendix

Ansatz (16) is obviously justified, because the expectation value $\langle\hat{A}\rangle$ of *any* property type A is given by

$$\langle\hat{A}\rangle = \text{Tr}(\hat{A}\rho). \tag{I}$$

One could, however, object that the preparation-induced change of the ensemble's statistical operator has to be reflected by the transformation

$$\rho_{\text{total}} \to \frac{\text{Tr}_{\text{C,A}}\Big((\hat{P}\otimes\hat{1}_{\text{B}})\rho_{\text{total}}(\hat{P}\otimes\hat{1}_{\text{B}})\Big)}{\text{Tr}_{\text{total}}\Big((\hat{P}\otimes\hat{1}_{\text{B}})\rho_{\text{total}}(\hat{P}\otimes\hat{1}_{\text{B}})\Big)} \tag{II}$$

instead. For the numerator $\text{Tr}_{\text{C,A}}(N)$ of the right hand side of (II) we obtain

$$\begin{aligned} N \;=\; & \frac{1}{8}\Big((\hat{C}_{11}\otimes\hat{A}_{11}+\hat{C}_{12}\otimes\hat{A}_{12}+\hat{C}_{21}\otimes\hat{A}_{21}+\hat{C}_{22}\otimes\hat{A}_{22}) \\ & \otimes(c_{11}\hat{B}_{22}-c_{12}\hat{B}_{21}-c_{21}\hat{B}_{12}+c_{22}\hat{B}_{11})\Big)\,. \end{aligned} \tag{III}$$

$$\Rightarrow \text{Tr}_{\text{C,A}}(N) = \frac{1}{4}\left(c_{11}\hat{B}_{22}-c_{12}\hat{B}_{21}-c_{21}\hat{B}_{12}+c_{22}\hat{B}_{11}\right) \tag{IV}$$

Formation of the total trace yields the value $1/4\,(c_{11}+c_{22}) = 1/4$ so that ρ_{total} is finally mapped onto

$$c_{11}\hat{B}_{22}-c_{12}\hat{B}_{21}-c_{21}\hat{B}_{12}+c_{22}\hat{B}_{11} = \sigma_3\sigma_1\rho_{\text{B}}\sigma_1\sigma_3 \tag{V}$$

which is exactly the result (20) obtained in the usual way.

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